INSTRUMENTATION ENGINEERING

ELECTRICAL MACHINES

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Electrical Machines

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Electrical Machines

CHAPTER 5

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Magnetic Circuits

Introduction

The electromagnetic system is an essential element of all rotating electric machinery, electromechanical devices and static devices like the transformer. The role of electro-magnetic system is to establish and control electromagnetic fields for carrying out conversion of energy, its processing and transfer. Electromechanical energy conversion takes place via the medium of a magnetic field or electrical field, but most practical converters use magnetic field as the coupling medium between electrical and mechanical systems. In transformers, the electrical energy is transferred from one electrical circuit to another electrical circuit via the medium of a magnetic field as the coupling medium between two electrical circuits. This is due to fact that the energy storing capacity of magnetic field is much greater than that of the electric field.

1.1   MAGNETIC CIRCUITS

- The complete closed path followed by the lines of flux is called a magnetic circuit. In low power electrical machines, magnetic field is produced by permanent magnets. But in high-power electrical machinery and transformers, coupling magnetic field is produced by electric current.

Fig.: Magnetic circuit

- In a magnetic circuit, the magnetic flux is due to the presence of a magnetomotive force same as in an electric circuit, the current is due to the presence of a electromotive force.
- The mmf is created by a current flowing through one or more turns.

$$MMF = \text{Current} \times \text{Number of turns in the coil}$$

$$f = MMF = NI \text{ (ampere-turns) or (ATs)}$$

- The magnetic flux ϕ may be defined as the magnetomotive force per unit reluctance.

$$\phi = \frac{MMF}{\text{Reluctance}}$$

where reluctance in magnetic circuit is similar as resistance in electric circuit.

- The opposition offered to the magnetic flux is called reluctance,

$$Rl = \frac{l}{\mu A} \text{ AT/wb}$$
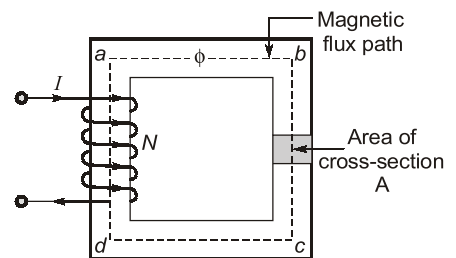
where, l = length of the magnetic path; A = area of cross-section normal to flux path, m^2.

μ = $\mu_0 \cdot \mu_r$ = permeability of the magnetic material

μ_r = relative permeability of magnetic material; μ_0 = permeability of free space = $4\pi \times 10^{-7}$ H/m.

The permeance of a magnetic circuit is the reciprocal of its reluctance.

$$P = \frac{1}{R_l}$$

Reluctances in a magnetic circuit obey the same rules as resistances in an electric circuit. The equivalent reluctance of a number of reluctances in series is the sum of the individual reluctances :

$$R_{l_{eq}} = R_{l_1} + R_{l_2} + R_{l_3} + ...$$

Permeances in series and parallel obey the same rules as electrical conductances.

1.1.1 Core with Air-Gap

A typical magnetic circuit with an air-gap is shown below in the figure. It is assumed that the air-gap is narrow and the flux coming out of the core passes straightly through the air-gap, such that the flux density in the air-gap is the same as in the core and core permeability μ is regarded as constant (linear magnetization). But in reality the flux in the air-gap fringes out so that the air-gap flux density is somewhat less that of the core (fringing effect).
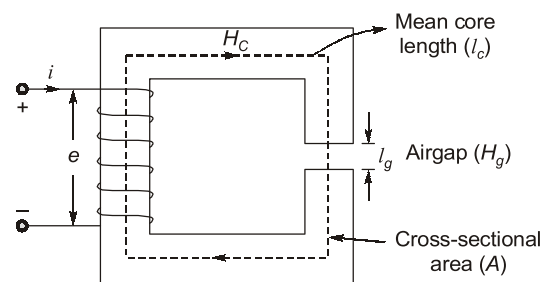
MMF Ni is now given as : $Ni = H_c l_c + H_g l_g = \dfrac{B_c}{\mu_c} l_c + \dfrac{B_g}{\mu_o} l_g$

Assuming that all the core flux passes straight down the air-gap (it means no fringing),

$$B_c = B_g$$

$\therefore$ $\qquad \phi = B_c A = B_g A$

$\therefore$ $\qquad N_i = \phi \left(\dfrac{l_c}{\mu_c A} \right) + \phi \left(\dfrac{l_g}{\mu_o A} \right) \quad \Rightarrow \quad N_i = \phi (R_{l_c} + R_{l_g})$

where, $\qquad R_{l_c} = \dfrac{l_c}{\mu_c \cdot A}$ = core reluctance ; $\qquad R_{l_g} = \dfrac{l_g}{\mu_o A}$ = air-gap reluctance

The magnetic energy stored in the air-gap and the iron core is given as

$$W = \frac{1}{2} L I^2 = \frac{1}{2} \left(\frac{N^2}{\text{Reluctance}} \right) \times I^2 = \frac{1}{2 \times \text{Reluctance}} \times (\phi \times \text{Reluctance})^2 = \frac{1}{2} \phi^2 R_T$$

where R_T is total reluctance that is $R_T = R_{l_c} + R_{l_g}$.

1.1.2 Fringing

At an air-gap in a magnetic core, the flux fringes out into neighbouring paths as shown in figure. The result is non-uniform flux density in the air-gap, enlargement of the effective air-gap area and a decrease in the average air-gap flux density.

If area of air-gap increases, then total area of core with consideration of air gap increases. Then average flux density gets reduced. It is possible to partially offset these inherent sources of error by using a "corrected" or "effective" mean path length and the cross-sectional area instead of actual physical length and area in the calculations. Satisfactory results may be achieved with this approximate method.
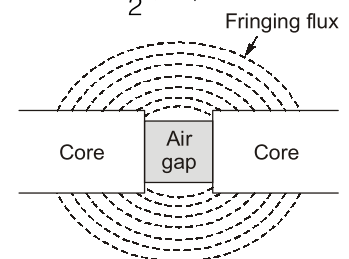
| EXAMPLE : 1.1 | An iron ring with a mean length of magnetic path of 20 cm and of small cross-section has an air gap of 1 mm. It is wound uniformly with a coil of 440 turns. A current of 1 A in the coil produces a flux density of $16\pi \times 10^{-3}$ Wb/m^2. Neglecting leakage and fringing, calculate the relative permeability of iron. |
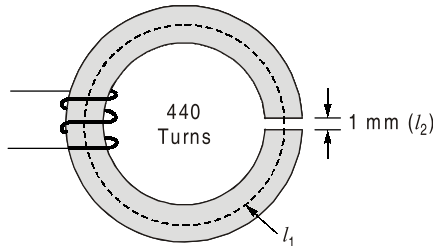
Solution :

The figure shows an iron ring of mean length

$$= 20 \text{ cm} = \ell_1$$

Length of air gap = 1 mm = 1×10^{-3} m = ℓ_2

Number of turns would = 440 turns = N

$$\text{Current in the coil} = 1 \text{ A} = I$$

$$\text{Flux density} = 16\pi \times 10^{-3} \text{ Wb/m}^2 = B$$
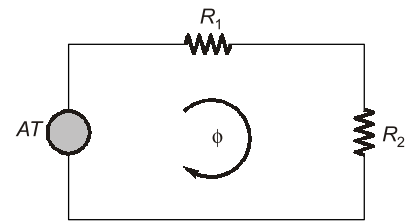
The electrical equivalent is as shown given figure.

Here, $\quad R_1 = $ Reluctance of iron; $\quad R_2 = $ Reluctance of air gap.

$$\therefore \quad AT = \phi(R_1 + R_2) \implies \phi = BA \ (A = \text{Area})$$

$$AT = BA\left(\frac{\ell_1}{\mu_0\mu_r A} + \frac{\ell_2}{\mu_0 A}\right) = \frac{B}{\mu_0}\left[\frac{\ell_1}{\mu_r} + \ell_2\right]$$

$$\therefore \quad \frac{\ell_1}{\mu_r} + \ell_2 = \frac{\mu_0 AT}{B} = \frac{4\pi \times 10^{-7} \times 440 \times 1}{16\pi \times 10^{-3}}$$

$$\frac{20 \times 10^{-2}}{\mu_r} + 1 \times 10^{-3} = 110 \times 10^{-4}$$

$$\therefore \quad \mu_r = \frac{20 \times 10^{-2}}{10 \times 10^{-3}} = 20$$

| EXAMPLE : 1.2 | A magnetic core is excited with two different arrangements of exciting coils as shown in figure. The resistance of the exciting coils is negligible. The same sinusoidal voltage at a specified frequency is applied to the exciting coil in each case. If the flux density and the exciting current in case (a) are $B = 0.1$ tesla and $I = 8$ A, calculate the values of these quantities in case (b). |

(a)   (b)

Solution :

(a)   (b)
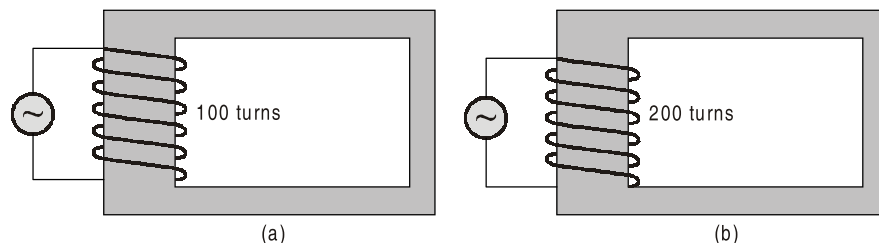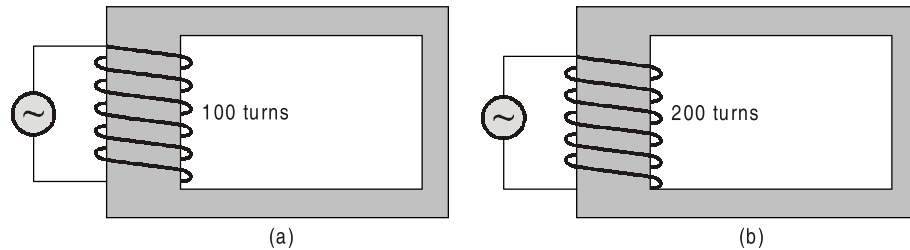
Method-1 : The resistance of exciting coil is negligible,

$$B_1 = 0.1 \text{ tesla}, I_1 = 8 \text{ A}$$

We know, $AT = \text{flux} \times \text{reluctance}$, $\therefore AT = \phi \times \dfrac{\ell}{\mu_0 \mu_r A} = B \times \dfrac{\ell}{\mu_0 \mu_r}$

Since $B = \dfrac{\phi}{A}$, $\qquad\qquad AT = \text{Ampere-turns} = I \times N \quad (N = \text{Number of turns})$

$\therefore \qquad\qquad\qquad I_1 N_1 = B_1 \times \dfrac{\ell}{\mu_0 \mu_r}$

Similarly, $\qquad\qquad I_2 N_2 = B_2 \times \dfrac{\ell}{\mu_0 \mu_r}$

$\therefore \qquad\qquad\qquad \dfrac{I_1 N_1}{I_2 N_2} = \dfrac{B_1}{B_2} \quad \Rightarrow \quad \dfrac{8 \times 100}{I_2 \times 200} = \dfrac{0.1}{B_2}$

$\therefore \qquad\qquad\qquad I_2 = 40 B_2$

$$\text{Induced emf} = -N\dfrac{d\phi}{dt}$$

$\therefore \qquad\qquad \dfrac{E_1}{E_2} = \dfrac{-N_1 \dfrac{d\phi_1}{dt}}{-N_2 \dfrac{d\phi_2}{dt}} = \dfrac{N_1}{N_2} \times \dfrac{\phi_1}{\phi_2} = \dfrac{N_1 B_1}{N_2 B_2}$

Since $E_1 = E_2$ (same), $\quad N_1 B_1 = N_2 B_2 \quad \Rightarrow \quad 100 \times 0.1 = 200 \times B_2$

$\therefore \qquad\qquad\qquad B_2 = \dfrac{100 \times 0.1}{200} = 0.05 \text{ tesla}$

$$I_2 = 40(0.05) = 2 \text{ A}$$

Method-2 : $\qquad\qquad L \propto N^2$

$$L_2 = 4L_1$$

$\Rightarrow \qquad\qquad\qquad X_2 = 4 X_1 \qquad\qquad (\because 2\pi f L_2 = 4(2\pi f L_1); X_2 = 4X_1)$

$\Rightarrow \qquad\qquad\qquad I_2 = \dfrac{1}{4} I_1 \qquad\qquad \left(\because \dfrac{E_1}{E_2} = \dfrac{I_1 X_1}{I_2 X_2} \text{ and } E_1 = E_2 \right)$

$$MMF_2 = \dfrac{1}{2} MMF_1$$

For same magnetic circuit, $\quad B_2 = \dfrac{1}{2} B_1 = \dfrac{0.1}{2} = 0.05 \text{ tesla} \qquad (\because MMF \propto \phi \propto \beta)$

$$I_2 = \dfrac{1}{4} \times I_1 = \dfrac{1}{4} \times 8 = 2 \text{ A} \qquad (\because MMF \propto \phi \propto \beta)$$

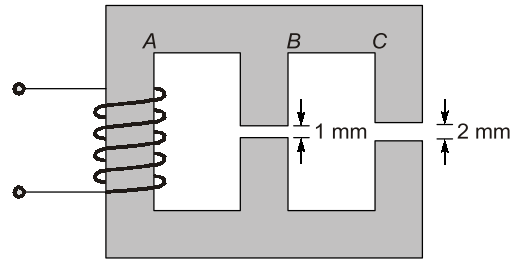| **EXAMPLE : 1.3** | In the magnetic circuit shown in figure, the areas of cross-section of limbs B and C are resp. 0.01 m² and 0.02 m². Air gaps of lengths 1.0 mm and 2.0 mm respectively are cut in the limbs B and C. If the magnetic medium can be assumed to have infinite permeability and the flux in limb B is 1.0 Wb. Find the flux in limb A? |

Solution :

Area of cross-section of limb B = 0.01 m²   Area of cross-section of limb C = 0.02 mm²

Air gap length = 1.0 mm for limb B Air gap length = 2.0 mm for limb C

Magnetic medium is assumed as infinite permeability

$\therefore$ Reluctance of iron path is zero since $R \propto \dfrac{1}{\mu}$

Flux in limb, $B = 1.0$ Wb
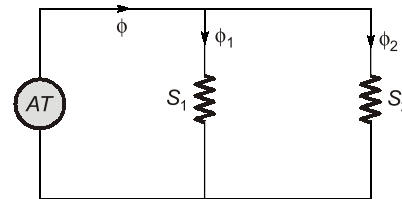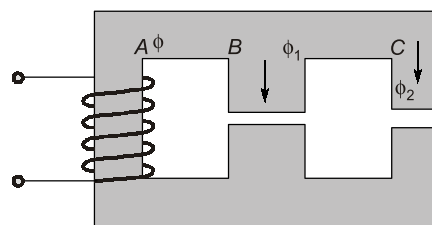
where, R_1 – reluctance of air gap of limb B; R_2 – reluctance of air gap of limb C

ϕ_1 – flux across air gap of limb B; ϕ_2 – flux across air gap of limb C

$\therefore \qquad\qquad\qquad\qquad R_1 \times \phi_1 = R_2 \times \phi_2$

$\therefore \quad \dfrac{\ell_1}{\mu_0 \times A_1} \times \phi_1 = \dfrac{\ell_2}{\mu_0 \times A_2} \times \phi_2 \quad \Rightarrow \quad \phi_2 = \dfrac{A_2}{A_1} \times \dfrac{\ell_1}{\ell_2} \times \phi_1 = \dfrac{0.02}{0.01} \times \dfrac{1}{2} \times 1 = 1\,\text{Wb}$

$\therefore \qquad\qquad$ Flux in limb $A = \phi_1 + \phi_2 = 1$ Wb + 1 Wb = 2 Wb

1.2   LEAKAGE FLUX

For an ideal magnetic circuit, we assume that the flux completely flow through the low reluctance path of iron. But in practical case, a small amount of flux produced by a coil or magnetic passes through region of space generally air spaces, electrical insulations, conductor of structural members of the system, rather than along the main path of the magnetic circuit. These flux lines are termed as leakage flux lines. In coupled circuit with two or more windings, the flux which links one coil does not link the other, is known as leakage flux. Its effect on the analysis of electrical machinery is carried out by replacing it by an equivalent leakage reactance
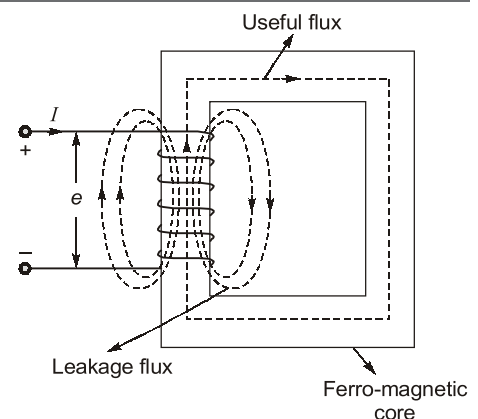
$$L = \frac{\text{Flux linkage}}{\text{Current}} = \frac{\lambda}{I} = \frac{N\phi}{I}$$

1.3 INDUCED EMF

1.3.1 Faraday's Law of Electromagnetic Induction

According to Faraday's law, whenever the magnetic flux linked with a circuit changes, an emf is induced in it, and magnitude of the induced emf is equal to the rate of change of flux-linkages.

$$e \propto \frac{d\varphi}{dt} \propto \frac{d(\phi N)}{dt}$$

$$e = -N\frac{d\phi}{dt}$$

where, e = induced emf; N = number of turns in the coil

φ = flux linkages with the coil ($N\phi$); t = time in seconds

'–' (minus) sign indicates the direction of induced emf which can be found by Lenz's law.

1.3.2 Lenz's Law

This law states that, electromagnetically induced current always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

1.3.3 Statically and Dynamically Induced EMF

- If the flux is time-varying in nature, then due to the inherent rate of change it automatically links stationary conductor and induces emf across it. This type of induced emf is called statically induced emf. e.g., transformer.

- If the flux is time-invarying in nature, then it requires a relative motion between flux and the conductor for effective rate of change of flux linkages to get induced emf. This type of induced emf is called dynamically induced emf is called dynamically induced emf. e.g., DC machine, synchronous machine.

EXAMPLE : 1.4

The magnetic circuit has dimensions $A_C = 4 \times 4$ cm^2, mean path length $l_c = 40$ cm, air-gap length $l_g = 0.06$ cm and number of turns, $N = 600$ turns. Assume value of $\mu_r = 6000$ for iron. Find the following :

(a) Induced emf for $B_c = 1.2 \sin 314t$ T.

(b) Core reluctance (R_c) and air-gap reluctance (R_g)

(c) Coil inductance (L)

Solution :

(a) $B_C = 1.2$ T, $A_C = 4 \times 4$ cm^2

$\phi = B_C A_C = 1.2 \times 4 \times 4 \times 10^{-4} = 19.2 \times 10^{-4}$ Wb

$\lambda = N\phi = 600 \times 19.2 \times 10^{-4} = 1.152$ Wb-turns

Emf is, $e = \frac{d\lambda}{dt} = \frac{1.152d}{dt}(\sin 3.4t) = 361.7 \cos 314t$ V

(b) $R_C = \frac{l_c}{\mu_o \mu_r A_c} = \frac{40 \times 10^{-2}}{4\pi \times 10^{-7} \times 6000 \times 4 \times 4 \times 10^{-4}} = 3.316 \times 10^4$

$R_g = \frac{l_g}{\mu_o A_g} = \frac{l_g}{\mu_o A_c} = \frac{6 \times 10^{-4}}{4\pi \times 10^{-7} \times 16 \times 10^{-4}} = 29.856 \times 10^4 \ (\because A_g = A_c)$

(c)
$$Ni = \frac{B_c}{\mu_o \mu_r} l_c + \frac{B_g}{\mu_o} l_g$$

$$i = \frac{B_c}{\mu_o N}\left(\frac{l_c}{\mu_r} + l_g\right) = \frac{1.2}{4\pi \times 10^{-7} \times 600}\left(\frac{40}{6000} + 0.06\right) \times 10^{-2} \quad (\because B_g = B_c)$$

$$= 1.06 \text{ A}$$

$$L = \frac{\lambda}{i} = \frac{1.152}{1.06} = 1.09 \text{ H}$$

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OBJECTIVE BRAIN TEASERS

Q.1 A cast steel electromagnet has an airgap of length 2 mm and an iron path of length 30 cm. The number of ampere turns necessary to produce a flux density of 0.8 Wb/m² in the gap is _____. Neglect leakage and fringing. (For 0.8 Wb/m² cast steel requires 750 AT/m).

Q.2 A cast steel ring has a circular cross-section 3 cm in diameter and a mean circumference of 80 cm. The ring is uniformly wound with 600 turns.

(a) The current required to produce a flux of 0.5 m Wb in the ring is _____ A.

(b) If a saw cut 2 mm wide is made in the ring, then approximately the flux produced by the current found in (a) is _____ mWb.

(c) The current value which will give the same flux as in (a), after the air gap of 2 mm is made in the ring is _____ A.

Assume the gap density to be the same as in the iron and neglect fringing.

(For 0.705 Wb/m² cast steel requires 670 AT/m).

Q.3 An iron ring of mean length 50 cm has an air gap of 1 mm and a winding of 200 turns. If the permeability of the iron is 300 and a current of 1 A flows through the coil, then the flux density is _____.

Q.4 Why the transformer stampings are varnished before being used to build the core?

(a) To increase air-gap between stampings

(b) To reduce hysteresis loss

(c) To reduce eddy current loss

(d) To provide strength to the core

Q.5 Maximum flux established in an AC excited iron core is influenced by

(a) frequency only

(b) voltage only

(c) both voltage and frequency

(d) reluctance of the core

Q.6 A circular iron core has an air-gap cut in it and is excited by passing direct current through a coil wound on it. The magnetic energy stored in the air-gap and the iron core is

(a) in inverse ratio of their reluctance

(b) in direct ratio of their reluctances

(c) equally divided among then

(d) energy resides wholly in the iron core

Q.7 In the electromagnetic relay of given figure below the reluctance of the iron path is negligible. The coil self-inductance is given by the expression

(a) $\mu_0 N^2 a/x$ (b) $\mu_0 N/2 \ ax$
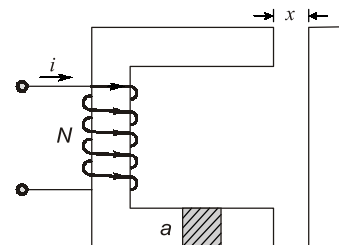
(c) $\mu_0 N^2 a/2x$ (d) $\mu_0 N^2/2 \ ax$

Q.8 An iron-cored choke with 1 mm air-gap length, draws 1 A when fed from a constant voltage AC

source of 220 V. If the length of air-gap is increased to 2 mm, the current drawn by the choke would

(a) become nearly one half
(b) remain nearly the same
(c) become nearly double
(d) become nearly zero

Q.9 The unit of inductance is

(a) Wb T/A
(b) V s/A
(c) H-turns^2
(d) All are equivalent

Q.10 Building steel core out of stampings reduces eddy current loss because,

(a) it increases core resistivity.
(b) it increases the effective length of eddy current paths thereby increasing effective resistance to the flow of eddy currents.
(c) it increases core permeability.
(d) it reduces the effective length of eddy current path, thereby reducing effective resistance to the flow of eddy currents.

Q.11 In a 4-pole dynamo, the flux/pole is 15 mWb. The average emf induced in one of the armature conductors, if armature is driven at 600 rpm

(a) 2.5 V
(b) 0.6 V
(c) 9 V
(d) 0.9 V

ANSWER KEY

1. (1498.24)   2. (Sol)   3. (Sol)   4. (c)
5. (c)   6. (b)   7. (c)   8. (c)   9. (d)
10. (b)   11. (b)

HINTS & EXPLANATIONS

1.   (1498.24)

Length of iron path,
$$l_i = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$$
Length of air gap = 2 mm = 2×10^{-3} m
$$B = NI \times \frac{\mu_o \mu_r}{l}$$

$$NI = \frac{Bl}{\mu_o \mu_r}$$

$$NI_{air} = \frac{0.8 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} = 1273.24 \text{ AT}$$

$$NI_{cast\ steel} = 750 \text{ AT/m} \times l_i$$
$$= 750 \times 30 \times 10^{-2} = 225 \text{ AT}$$

$$NI_{Total} = NI_{cast\ steel} + NI_{air}$$
$$= 1498.24 \text{ AT}$$

2.   (Sol)

(a) The cross-sectional area
$$= \frac{\pi d^2}{4} \times 10^{-4} = \frac{\pi \times 9}{4} \times 10^{-4}$$
$$= 7.068 \approx 7.1 \times 10^{-4} \text{ m}^2$$

The flux density,
$$B = \frac{\phi}{A} = \frac{0.5 \times 10^{-3}}{7.1 \times 10^{-5}} = \frac{5}{7.1}$$
$$= 0.705 \text{ Wb/m}^2$$

$\therefore$ AT required = $670 \times 0.8 = 536$

$\therefore$ The current required
$$= \frac{AT}{N} = \frac{536}{600} = 0.89 \text{ A}$$

(b) Flux without air gap = 0.5 mWb
$$\phi = \frac{NI}{S_i}$$

$$0.5 \times 10^{-3} = \frac{600 \times 0.8933}{S_i}$$

S_i = Reluctance of iron (steel) = 1072000
S_a = Reluctance of air
$$= \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times \left(\pi \times \frac{d^2}{4}\right)} = 2251581.9$$

S_T = Total reluctance
$$= S_i + S_a = 3323581.9$$
ϕ_{new} = new flux with air gap
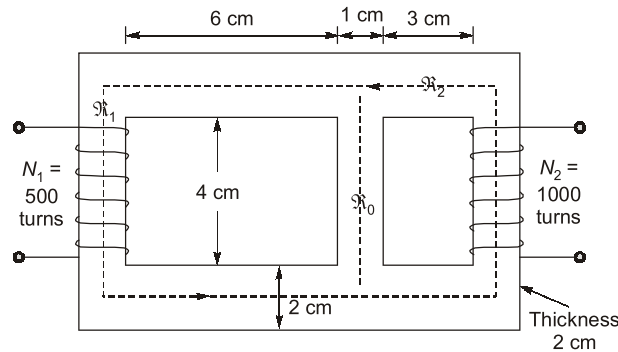$$\phi_{new} = \frac{536}{3323581.9} = 0.16127 \text{ mWb}$$

(c) Ampere turn required = NI
$$= \phi \times S_T \text{(Flux} \times \text{Total Reluctance)}$$

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CONVENTIONAL BRAIN TEASERS

Q.1 For the magnetic circuit of figure find the self and mutual inductances between the two coils. Core permeability = 1600.

1. (Sol)

$$l_1 = (6 + 0.5 + 1) \times 2 + (4 + 2) = 21 \text{ cm}$$

$$l_2 = (3 + 0.5 + 1) \times 2 + (4 + 2) = 15 \text{ cm}$$

$$l_0 = 4 + 2 = 6 \text{ cm}$$

$$R_1 = \frac{21 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 2 \times 2 \times 10^{-4}} = 0.261 \times 10^6$$

$$R_2 = \frac{15 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 2 \times 2 \times 10^{-4}} = 0.187 \times 10^6$$

$$R_0 = \frac{6 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 1 \times 2 \times 10^{-4}} = 0.149 \times 10^6$$

(i) Coil 1 excited with 1 A :

$$R = R_1 + R_0 \parallel R_2 = 0.261 + 0.1871 \parallel 0.149 = 0.344 \times 10^6$$

$$\phi_1 = \frac{(500 \times 1)}{(0.344 \times 10^6)} = 1.453 \text{ mWb}$$

By flux division (similar to current division) :

$$\phi_{21} = \phi_2 = \frac{1.453 \times 0.149}{(0.149 + 0.187)} = 0.64 \text{ mWb}$$

$$L_{11} = N_1\phi_1 = 500 \times 1.453 \times 10^{-3} = 0.7265 \text{ H}$$

$$M_{21} = N_2\phi_{21} = 1000 \times 0.649 \times 10^{-3} = 0.64 \text{ H}$$

(ii) Coil 2 excited with 1 A :

$$R = R_2 + \frac{(R_0 R_1)}{(R_0 R_1)} = \frac{0.187 + (0.149 \times 0.281)}{(0.149 + 0.281)} \times 10^6 = 0.284 \times 10^6$$

$$\phi_2 = \frac{(1000 \times 1)}{(0.284 \times 10^6)} = 3.52 \text{ mWb}$$

$$L_{22} = N_2\phi_2 = 1000 \times 3.52 \times 10^{-3} = 3.52 \text{ H}$$

$$M_{12} = M_{21} \text{ (bilateral)} = 0.65 \text{ H}$$